Tutorübung am 21.06.2011 - Mathematik für das Studium Naturale, SS2011
Blatt 17: Schwingungsgleichungen, Autor: Dominik Jüstel

restart :

Die Schwingungsgleichung lautet
my''(t) + gam*y'(t) + ky(t) = f(t)
mit m,k > 0, gam >= 0.

Initialisierung der Parameter m, gam und k
(m = 1, gam = 0.1, k = 1)

$m := 1;$
$gam := 0.1;$
$k := 1;$

$$1$$
$$0.1$$
$$1 \qquad \textbf{(1)}$$

Berechnung der Diskriminante

$$Dis := \left(\frac{gam}{2\,m}\right)^2 - \frac{k}{m};$$

Berechnung der Lösungen der homogenen Gleichung

$$\lambda_0 := -\frac{gam}{2\,m}\ :$$

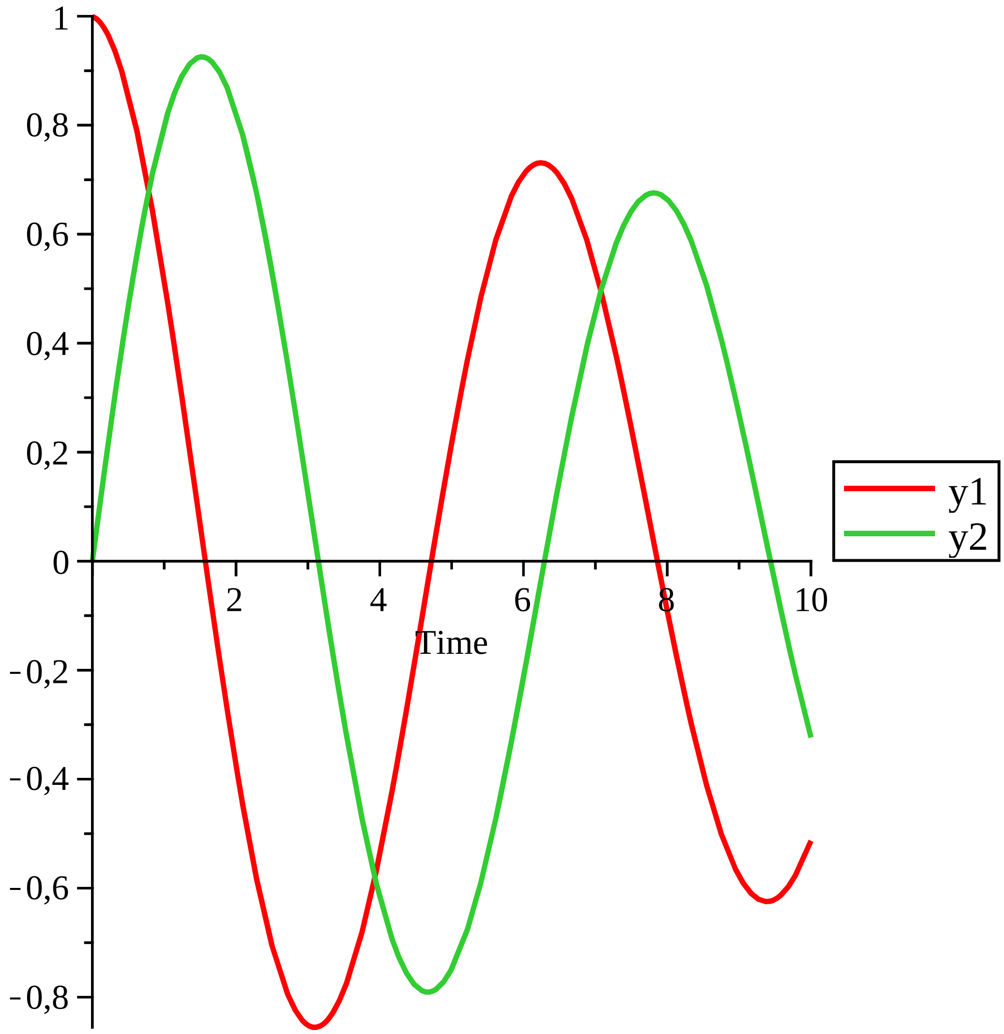
if $Dis > 0$ **then** #\` \`***starke Dämpfung***
  $\lambda_+ := \lambda_0 + \mathrm{sqrt}(Dis)$:
  $\lambda_- := \lambda_0 - \mathrm{sqrt}(Dis)$:
  $y1 := t \to \exp(\lambda_+ \cdot t);$
  $y2 := t \to \exp(\lambda_- \cdot t);$
elif $Dis = 0$ **then** #\` \`***kritische Dämpfung***
  $y1 := t \to \exp(\lambda_0 \cdot t);$
  $y2 := t \to t \cdot \exp(\lambda_0 \cdot t);$
else #\` \`***schwache Dämpfung***
  $\omega := \mathrm{sqrt}(-Dis)$:
  $y1 := t \to \exp(\lambda_0 \cdot t) \cdot \cos(\omega \cdot t);$
  $y2 := t \to \exp(\lambda_0 \cdot t) \cdot \sin(\omega \cdot t);$
end;

$$0.9987492178$$
$$t \to e^{\lambda_0 t} \cos(\omega\, t)$$

$$t \rightarrow e^{\lambda_0 t} \sin(\omega t) \tag{2}$$

Plot der Lösungen

plot([*y1*(*t*), *y2*(*t*)], *t* = 0 ..10, *legend* = ["y1", "y2"], *legendstyle* = [*location* = *right*], *thickness* = 2,
labels = ["Time", ""]);



Initialisierung der Anfangsbedingungen
y(0) = y0, y'(0) = v0.
(y0 = 0, v0 = 0)

y0 := 0;
v0 := 0;

$$0$$
$$0 \tag{3}$$

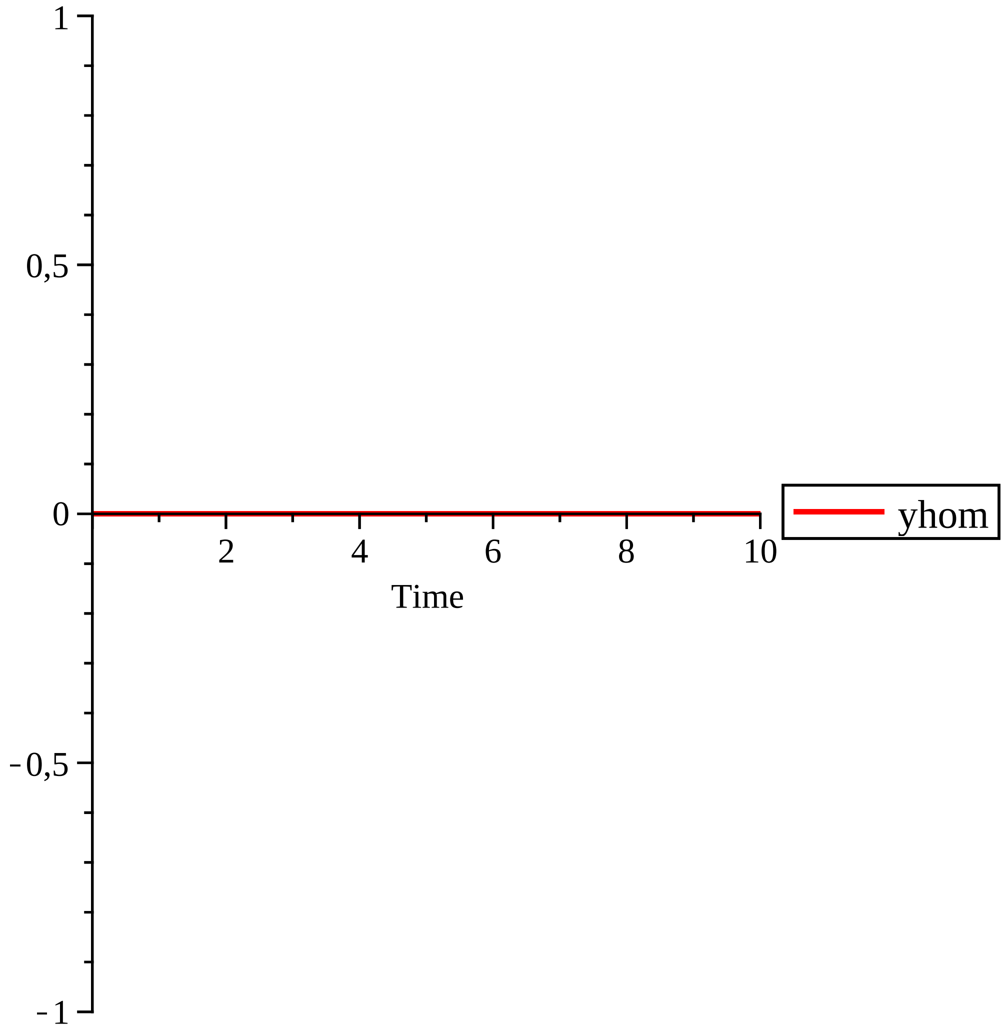
Lösen des homogenen Anfangswertproblems

yhom := *t*→*A*·*y1*(*t*) + *B*·*y2*(*t*);
param := *solve*([*yhom*(0) = *y0*, D(*yhom*)(0) = *v0*], [*A*, *B*]);
A := *rhs*(*param*[1, 1]);
B := *rhs*(*param*[1, 2]);

$$t \to A\,y1(t) + B\,y2(t)$$
$$[[A = 0., B = 0.]]$$
$$0.$$
$$0. \qquad \textbf{(4)}$$

Plot der homogenen Lösung

$plot(yhom(t), t = 0 ..10, legend = [\text{"yhom"}], legendstyle = [location = right], thickness = 2, labels = [\text{"Time"}, \text{""}]);$



Initialisierung der harmonischen Anregung
f(t) = Amp*cos(omega0*t)
(Amp = 1, omega0 = 2)

$Amp := 1;$
$\omega 0 := 2;$
$f := t \to Amp \cdot \cos(\omega 0 \cdot t);$

$$1$$
$$2$$

$$t \to Amp \cos(\omega 0\ t) \quad (5)$$

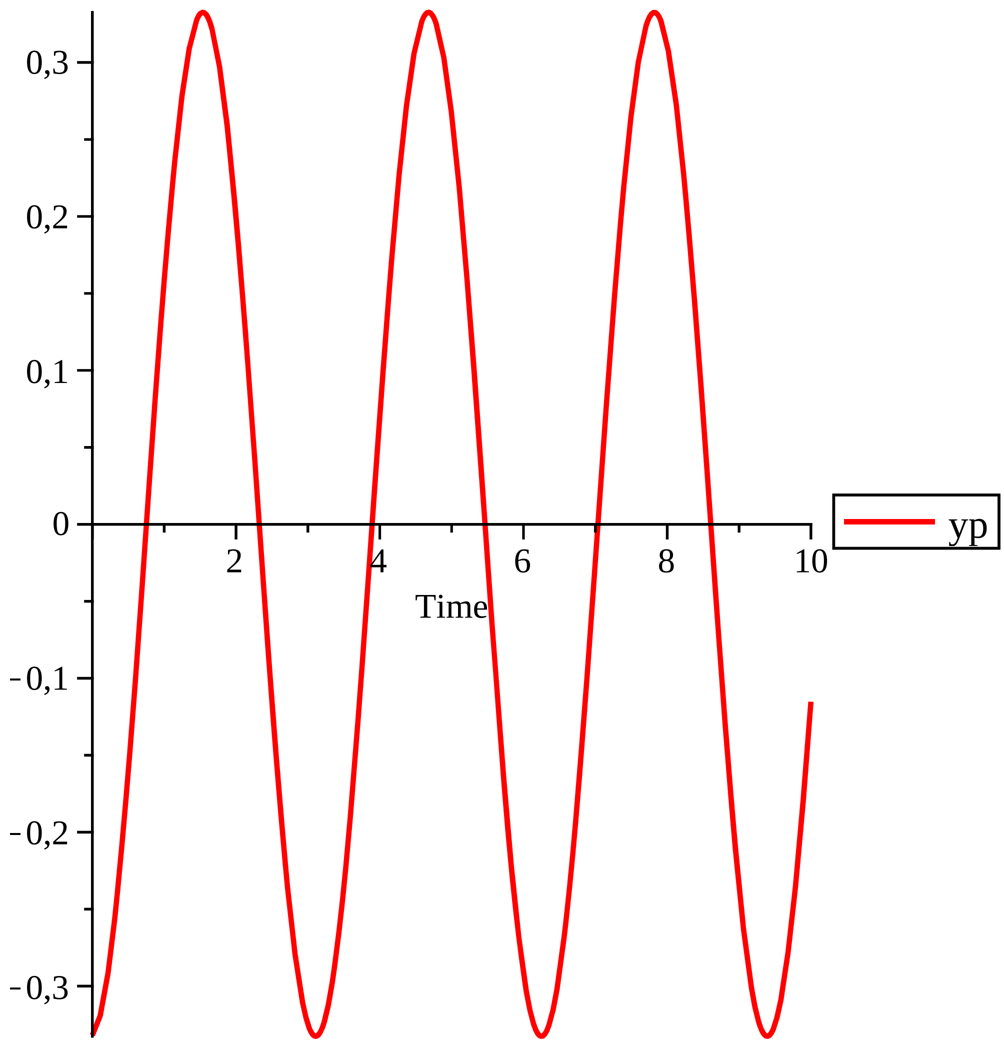
Spezielle Lösung yp(t) = sigma cos(omega_0 t+delta) (hergeleitet in T17.1)
(Verwenden Sie für einen Ausdruck der Form arctan(a/b) die Funktion arctan(a,b).)

$$c1 := \frac{Amp \cdot (k - m \cdot \omega 0^2)}{(k - m \cdot \omega 0^2)^2 + gam^2 \cdot \omega 0^2};$$
$$c2 := \frac{Amp \cdot gam \cdot \omega 0}{(k - m \cdot \omega 0^2)^2 + gam^2 \cdot \omega 0^2};$$
$$\delta := \arctan(c2, c1);$$
$$\sigma := \frac{c1}{\cos(\delta)};$$
$$yp := t \to \sigma \cdot \cos(\omega 0 \cdot t - \delta);$$

$$-0.3318584071$$
$$0.02212389381$$
$$3.075024490$$
$$0.3325950526$$
$$t \to \sigma \cos(\omega 0\ t - \delta) \quad (6)$$

Plot der speziellen Lösung

$plot(yp(t), t = 0..10, legend = ["yp"], legendstyle = [location = right], thickness = 2, labels = ["Time", ""]);$



Lösung des inhomogenen Anfangswertproblems und Plot der Lösung

$yinhom := t \rightarrow C1 \cdot y1(t) + C2 \cdot y2(t) + yp(t);$
$param := solve([yinhom(0) = y0, \mathrm{D}(yinhom)(0) = v0], [C1, C2]);$
$C1 := rhs(param[1, 1]);$
$C2 := rhs(param[1, 2]);$

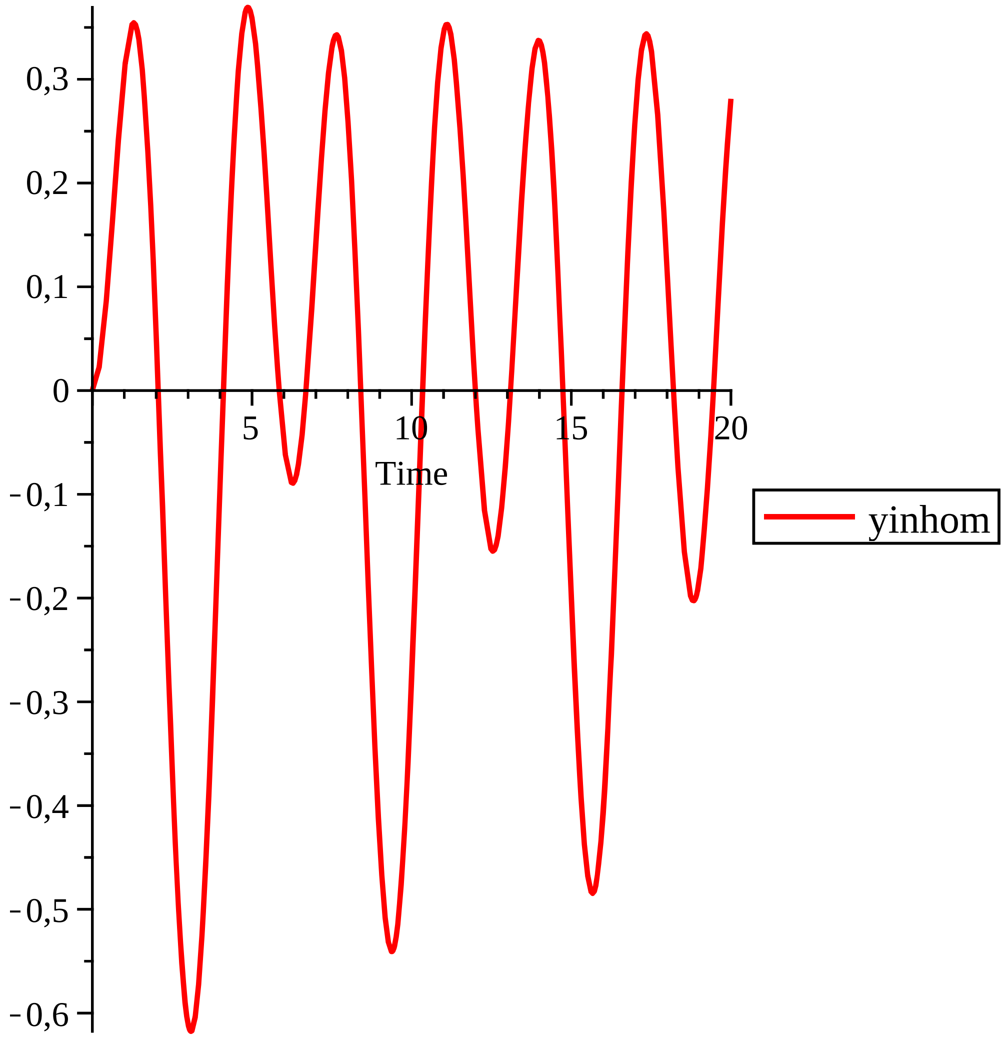
$$t \rightarrow C1\, y1(t) + C2\, y2(t) + yp(t)$$

$$[[C1 = 0.3318584071, C2 = -0.02768950067]]$$

$$0.3318584071$$

$$-0.02768950067 \quad \textbf{(7)}$$

$plot(yinhom(t), t = 0..20, legend = [\text{"yinhom"}], legendstyle = [location = right], thickness = 2, labels = [\text{"Time"}, \text{""}]);$



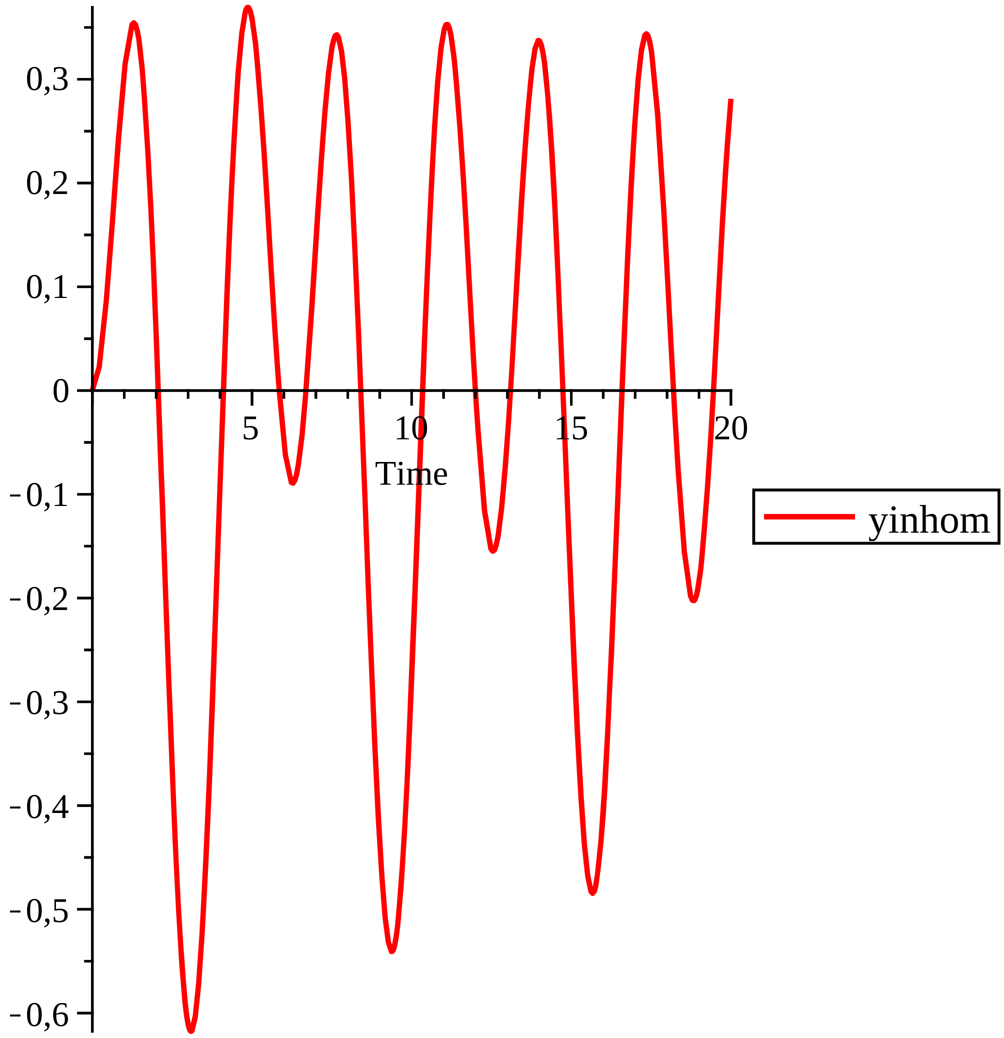
Verifizierung der Lösung mittels 'dsolve'

inhomSchwingung := *m*·*diff* (*diff* (*y*(*t*), *t*), *t*) + *gam*·*diff* (*y*(*t*), *t*) + *k*·*y*(*t*) = *f* (*t*);
AnfBed := *y*(0) = *y0*, D(*y*)(0) = *v0*;
solution := *dsolve*({*inhomSchwingung*, *AnfBed*});
plot(*rhs*(*solution*), *t* = 0 ..20, *legend* = ["yinhom"], *legendstyle* = [*location* = *right*], *thickness* = 2, *labels* = ["Time", ""]);

$$\frac{d^2}{dt^2}\, y(t) + 0.1 \left(\frac{d}{dt}\, y(t) \right) + y(t) = \cos(2\, t)$$

$$y(0) = 0,\ D(y)(0) = 0$$

$$y(t) = -\frac{125}{90174}\, e^{-\frac{1}{20}\, t} \sin\left(\frac{1}{20} \sqrt{399}\, t \right) \sqrt{399} + \frac{75}{226}\, e^{-\frac{1}{20}\, t} \cos\left(\frac{1}{20} \sqrt{399}\, t \right) - \frac{75}{226} \cos(2\, t) + \frac{5}{226} \sin(2\, t)$$

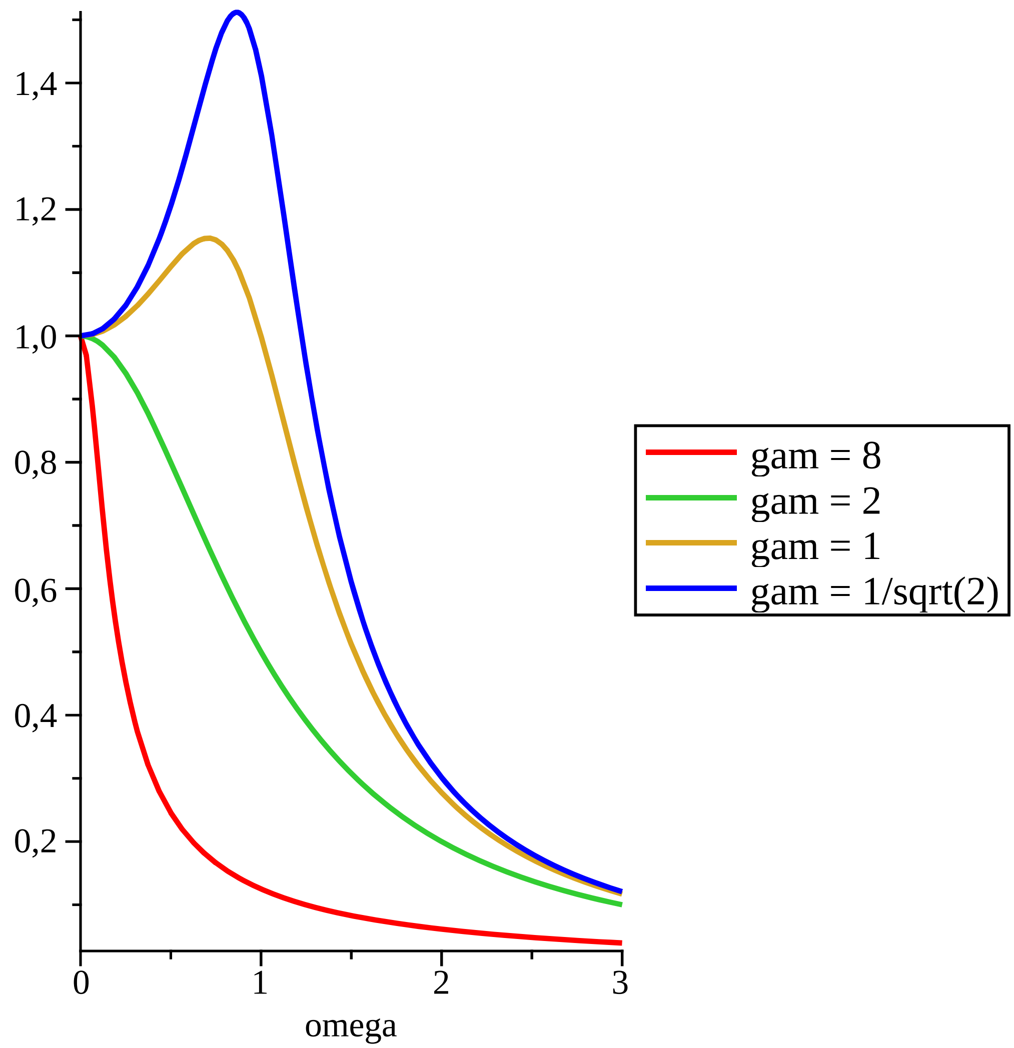


Plot der Resonanzkurve für verschiedene Werte von gam
(gam = 8, 2, 1, 1/sqrt(2))

$$\phi := (om, gam) \rightarrow \frac{1}{\text{sqrt}\left(\left(k - m \cdot om^2\right)^2 + gam^2 \cdot om^2 \right)};$$

$plot\left(\left[\phi(om, 8), \phi(om, 2), \phi(om, 1), \phi\left(om, \frac{1}{\text{sqrt}(2)}\right) \right], om = 0..3, legend = [\text{"gam = 8"}, \text{"gam = 2"}, \text{"gam = 1"}, \text{"gam = 1/sqrt(2)"}], legendstyle = [location = right], thickness = 2, labels = [\text{"omega"}, \text{""}] \right);$

$$(om, gam) \rightarrow \frac{1}{\sqrt{\left(k - m\, om^2\right)^2 + gam^2\, om^2}}$$


1,4
1,2
1,0
0,8
0,6
0,4
0,2
0
1
2
3
omega
gam = 8
gam = 2
gam = 1
gam = 1/sqrt(2)