

Exercise 1

Prove that if $n \in \mathbb{Z}$ the only solutions of the differential equation $r^2 F''(r) + r F'(r) - n^2 F(r) = 0$ which are twice differentiable for $r > 0$, are given by a linear combination of r^n and r^{-n} when $n \neq 0$, and of 1 and $\log r$ when $n = 0$.

Problem 2 (Harmonic functions on the disk)

$$(P) \begin{cases} \Delta u = 0 & \text{on } D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\} \\ u = f & \text{on } \partial D \end{cases}$$

• Here $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

Find (formally) a solution u of (P)

using polar (r, θ) coordinates and the method of separation of variables.